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LOGICAL PARADOXES IN LAW APPLICATION: FORMAL RATIONALITY VS PRINCIPLES OF JUSTICE

The article examines the problem of logical paradoxes arising in legal application processes due to conflicts between the formal rationality of legal norms and principles of justice. The relevance of this research is determined by the increasing complexity of legal systems, the development of analytical jurisprudence, and the necessity to reconsider the boundaries of applying formal-logical approaches in contemporary legal science and practice. A systematic analysis of logical paradoxes embedded in the structure of legal norms and judicial procedures has been conducted to clarify the fundamental limits of formal logic in jurisprudence and theoretically substantiate the need to turn to broader principles of justice when resolving legal conflicts. The research employs methods of formal-logical analysis, structural-functional approach, comparative-legal method, and hermeneutical approach to the interpretation of legal norms and precedents.

Classical paradoxes of legal application are analyzed: the paradox of norm self-application (using the example of appellate appeal), the paradox of presumption of innocence in the context of preventive measures, the paradox of limitation periods under conditions of technological progress, and the paradox of fair punishment as a conflict between individualization and equality. Specific manifestations of logical contradictions in international public law are examined, particularly the chronological paradox of customary international law formation and the doctrinal paradox of collegial adjudication in international judicial institutions. The connection between Gödel's incompleteness theorem and legal systems has been confirmed, representing an original authorial contribution to legal epistemology. The application of mathematical logic to the analysis of legal phenomena opens new perspectives for understanding structural limitations of normative systems. The work synthesizes classical works of the positivist school with critical approaches of contemporary research, providing a balanced theoretical foundation.

It is proven that logical paradoxes are not technical defects of legal technique, but fundamental characteristics of any complex normative systems, reflecting the principled impossibility of complete formalization of law according to Gödel's incompleteness theorem. It is established that principles of justice function as meta-norms that ensure structural integrity of the legal system under conditions of logical contradictions and allow law to remain a rational instrument of social regulation without being limited exclusively to formal logic.

Ключові слова*: logic of law, logical paradoxes, liar paradox, legal application, formal rationality, principles of justice, international law, Gödel's incompleteness theorem, legal hermeneutics.*

Ковальчук О.

Логічні парадокси у правозастосуванні: формальна раціональність vs принципи справедливості

У статті досліджується проблема логічних парадоксів, що виникають у процесі правозастосування внаслідок конфлікту між формальною раціональністю правових норм та принципами справедливості. Актуальність дослідження зумовлена зростаючою складністю правових систем, розвитком аналітичної юриспруденції та необхідністю переосмислення меж застосування формально-логічного підходу в сучасній правовій науці та практиці. Проведено системний аналіз логічних парадоксів, вбудованих у структуру правових норм і судових процедур, для з'ясування фундаментальних меж формальної логіки в юриспруденції та теоретичного обґрунтування необхідності звернення до ширших принципів справедливості при вирішенні правових колізій. У дослідженні застосовано методи формально-логічного аналізу, структурно-функціональний підхід, порівняльно-правовий метод та герменевтичний підхід до інтерпретації правових норм і прецедентів.

Проаналізовано класичні парадокси правозастосування: парадокс самозастосування норм (на прикладі апеляційного оскарження), парадокс презумпції невинуватості в контексті запобіжних заходів, парадокс строків давності в умовах технологічного прогресу та парадокс справедливого покарання як конфлікт між індивідуалізацією та рівністю. Розглянуто специфічні прояви логічних суперечностей у міжнародному публічному праві, зокрема хронологічному парадоксу формування звичаєвих норм міжнародного права та доктринальному

парадоксу колегіального судочинства в міжнародних судових інституціях. Підтверджено зв'язок між теоремою Геделя про неповноту та правовими системами, що становить оригінальний авторський внесок у правову епістемологію. Застосування математичної логіки до аналізу правових явищ відкриває нові перспективи для розуміння структурних обмежень нормативних систем. У роботі синтезовано класичні роботи позитивістської школи із критичними підходами сучасних досліджень, що забезпечило збалансовану теоретичну основу.

Доведено, що логічні парадокси є не технічними недоліками юридичної техніки, а фундаментальними характеристиками будь-яких складних нормативних систем, що відображають принципову неможливість повної формалізації права згідно з теоремою Геделя про неповноту. Встановлено, що принципи справедливості функціонують як метанорми, що забезпечують структурну цілісність правової системи в умовах логічних суперечностей та дозволяють праву залишатися раціональним інструментом соціального регулювання, не обмежуючись виключно формальною логікою.

***Ключові слова**: логіка права, логічні парадокси, парадокс брехуна, правозастосування, формальна раціональність, принципи справедливості, міжнародне право, Теорема Геделя про неповноту, юридична герменевтика.*

Statement of the problem. In contemporary jurisprudence, the question of the limits of formal logic as a tool for legal application arises increasingly often. On one hand, the legal system appeals to rationality, structure, and internal consistency – characteristics inherent to formal logic [1]. On the other hand, law performs not only a regulatory but also a humanistic function, which requires consideration of moral, ethical, and contextual factors (principles of justice). The tension between these two dimensions (formal rationality and the pursuit of justice) manifests in numerous legal conflicts, interpretations, and judicial precedents.

This antagonism becomes particularly acute in situations where the very structure of a norm becomes a source of logical paradox. A vivid example is the so-called «liar's paradox» in a legal context [2]. These are norms that have a self-referential character (the concept refers to itself) and create logical uncertainty [3]. For instance, statements like «do not obey this law» or legal constructions that regulate themselves call into question the very possibility of unambiguous application of such prescriptions. Such paradoxes not only cause theoretical difficulties but also have practical consequences for the functioning of the legal system, particularly regarding the stability of norms, predictability of decisions, and legitimacy of judicial procedures.

The issue of interaction between formal logic and principles of justice in legal application is the subject of intensive research in legal philosophy, analytical jurisprudence, and legal hermeneutics. In his works, H. L. A. Har emphasized the importance of clear rules and predictability of the legal system, highlighting the role of primary and secondary norms in ensuring the logical structure of the legal order [4]. Joseph Raz developed the concept of legal authority, also emphasizing rationality and consistency of legal thinking [5]. Meanwhile, R. Dworkin criticized legal positivism for excessive formalism, arguing that judges should be guided not only by the letter of the law but also by moral principles underlying the legal system. His thesis of «law as integrity» is based on the conviction that a just decision is impossible without moral evaluation of legal norms [6]. Representatives of critical legal studies, particularly Duncan Kennedy and Roberto Unger, illuminate how formal logic can conceal social prejudices and reinforce inequality [7; 8]. In the field of logic of law, important contributions were made by G. H. von Wright [9] and R. Alexy, the latter of whom proposed a logically grounded model of balancing principles in the process of legal argumentation [10]. Alongside this, researchers such as S. Shapiro consider law as an institutional fact, which presupposes the integration of logical rules with social conditions of their application [11].

In contemporary scholarship, several approaches have been formed to overcome the conflict between logical consistency and the demands of justice, but the question of internal paradoxes of legal thinking remains open and requires further theoretical understanding [12–15].

The research aims to investigate how logical paradoxes embedded in the structure of legal norms or judicial procedures reveal the limits of formal logic in jurisprudence and emphasize the necessity of turning to broader principles of justice in legal application.

Presentation of the main research material. Law is traditionally perceived as a system of logically structured norms based on principles of formal rationality [16]. However, legal practice demonstrates numerous cases where literal application of legal norms leads to results that contradict basic notions of justice [17]. These situations are not mere deficiencies in legislation. They reflect fundamental logical paradoxes embedded in the very structure of legal systems. A logical paradox in law arises when consistent application of formal-logical rules leads to conclusions that contradict either common sense, basic principles of justice, or other legal norms of the same system [18].

A classic example is the paradox of self-application of norms [19]. For instance, a norm that establishes: "All court decisions are subject to appellate review". Is the decision of an appellate court to refuse consideration of an appeal based on this same norm subject to review? Formal logic creates an infinite regress, while practical necessity demands the establishment of exceptions.

Formal methods of mathematics, particularly logical theorems, can reveal the limitations of complex normative systems, since legal norms, like mathematical axiomatics, are subject to the rules of formal logic and structural proof. In particular, Gödel's incompleteness theorem has direct application to law. One of the key ideas of Gödel's proof is the concept of self-reference [20]. He constructed a statement that essentially says: «This statement cannot be proven within the formal system». If the statement were provable, it would be false, leading to a contradiction. On the other hand, if the statement were unprovable, it would be true, demonstrating the existence of an unprovable true statement. Any sufficiently complex legal system cannot be simultaneously complete (capable of resolving all possible disputes) and consistent (free from internal conflicts of norms). This paradox of completeness and consistency of the legal system manifests in collisions between legal norms of different levels. For example, the constitutional principle of equality may contradict special norms that establish privileges for certain categories of persons [21]. A formal-logical approach cannot resolve such collisions without recourse to value judgments.

Another classic example is the paradox of presumption of innocence [22]. The presumption of innocence requires considering a person innocent until a court verdict is rendered. Simultaneously, the system of preventive measures provides for the possibility of restricting a person's rights at the stage of pre-trial investigation. This creates a logical contradiction: the system simultaneously considers a person innocent and acts as if they are potentially guilty.

The institution of limitation periods (a system of legal norms that regulate periods after which certain rights and obligations cease or change) is based on the presumption that with the passage of time, establishing truth becomes impossible. However, in cases where the truth is established with certainty (for example, thanks to new technologies), the formal application of limitation periods may lead to impunity for the guilty person. This manifests the paradox of limitation periods [23].

The principle of individualization of punishment requires consideration of all circumstances of a particular case. Simultaneously, the principle of equality before the law requires equal treatment of similar cases. These two principles create a contradiction: the more individualization, the less equality is ensured, and vice versa. This is the paradox of just punishment [24].

One of the most illustrative spheres where the limits of formal logic in jurisprudence are revealed is international law [25]. In particular, two types of logical paradoxes: the chronological paradox of formation of customary norms (customary international law) and the doctrinal paradox in collegial adjudication, illustrate the deep tension between the letter of the law and the spirit of justice. These paradoxes not only challenge the structural integrity of law but also have practical consequences for its legitimacy and predictability

Chronological paradox of international customary law. For a new customary norm of international law to be recognized as binding, a combination of state practice and conviction of its legal obligatoriness (opinio juris) is necessary [26]. The problem arises when states are expected to act as if bound already at the stage of norm formation. But such behavior is logically impossible without prior conviction of legal obligatoriness, which, in turn, appears only when the norm already exists. Thus, a circular mechanism emerges in which conviction precedes actual norm-creation but cannot be formed without it. Resolution of this dilemma is possible through recourse to the principle of good faith. According to this approach, it is the good faith of state behavior that generates primary obligation, even in the absence of an established norm. States, acting in good faith, actually create a situation in which conviction of obligatoriness precedes completed norm-creation [27]. Thus, good faith performs the role of a logical bridge between practice and legal norm.

Doctrinal paradox in international adjudication. Another manifestation of logical inconsistency is observed in the work of collegial organs of international adjudication, particularly in the so-called doctrinal paradox [28]. This paradox consists in the fact that when voting on separate elements of a case (for example, facts, legal norms, jurisdictional issues), the majority may support opposite positions than when voting on the final decision. As a result, a logically contradictory decision is formed in which the conclusion does not follow from the partial votes. In international law, such a contradiction may lead to significant legal uncertainty. This is particularly important in arbitrations and cases of international courts, where the final decision has binding force and serves as a source of precedent. Similar paradoxes threaten confidence in the judicial process and necessitate consideration of vote aggregation methods during the procedure development stage [28].

The development of information technologies and artificial intelligence generates new types of logical paradoxes in legal application [29]. In particular, algorithmic decision-making in the legal sphere creates a paradox of automated justice: the system strives for objectivity through the elimination of the human factor, but simultaneously loses the ability for contextual understanding of justice. This is particularly evident in systems of automated sentencing or risk assessment, where the formal logic of the algorithm may lead to discriminatory results, despite formally neutral criteria [30–33].

Particular attention should be paid to paradoxes arising in procedural law. The adversarial principle presupposes that truth is born in the dispute of parties; however, this creates a paradox of procedural truth: the system is directed toward establishing factual truth through a procedure that by its nature may distract from it. Parties are interested in victory, not in the objective establishment of facts, which may lead to situations where a procedurally correct decision is factually inaccurate [34; 35].

In the sphere of constitutional law, a paradox of democratic legitimacy is observed: constitutional courts, whose composition is not formed democratically, have the right to overturn decisions of democratically elected bodies of power [36]. This creates tension between the principle of popular sovereignty and the principle of the rule of law. Formal-logical justification of such a system faces the problem of self-legitimation: who gives an undemocratic body the right to limit democracy?

An analysis of logical paradoxes in law reveals the fundamental limits of formal rationality in jurisprudence. Even the most precisely formulated norm requires interpretation in the context of a specific situation. The process of interpretation always goes beyond pure logic. Moreover, it is impossible to foresee all possible life situations in the text of the law. Gaps in law are inevitable, and filling them requires recourse to principles that cannot be reduced to formal logic. Legal norms reflect different, sometimes incompatible values. Resolving conflicts between them requires value choice, which cannot be reduced to logical operations.

Contemporary research in the field of legal logic moves toward developing non-fundamental logical systems capable of working with incompleteness and contradiction. Basic laws of logic and fuzzy sets offer tools for formalizing legal reasoning under conditions of uncertainty. However, the question remains open: can such systems preserve the normative force of law without transforming into technical instruments devoid of moral content?

Conclusion. Logical paradoxes in legal application reflect fundamental tension between formal rationality and principles of justice, embedded in legal systems. Gödel's incompleteness theorem is confirmed in law: complex legal systems cannot be simultaneously complete and consistent. The process of legal application inevitably goes beyond deductive derivation of decisions. Interpretation of norms and resolution of collisions requires recourse to value judgments and moral criteria. Principles of justice function as meta-norms that ensure the integrity of the legal system under conditions of logical paradoxes.

Formal rationality is necessary but insufficient for just legal application. The emergence of algorithmic decision-making systems in jurisprudence introduces new dimensions to this paradox, as artificial intelligence may amplify formal logic while potentially diminishing contextual understanding of justice. This technological evolution demands careful consideration of how automated systems can preserve the moral dimension of legal reasoning. Legal paradoxes are not merely theoretical curiosities but practical challenges that require sophisticated responses from legal practitioners and theorists. Constitutional courts worldwide increasingly face situations where formal constitutional interpretation conflicts with evolving social values, highlighting the need for interpretative methodologies that can accommodate both logical consistency and moral progress. In international law, it is advisable to strengthen the significance of the principle of good faith and standardize decision-making methods to avoid paradoxical results. The development of paraconsistent logical frameworks may offer new tools for managing contradictions within legal systems without abandoning rational discourse entirely.

Legal education and practice require rethinking: law is a complex institutional practice that combines logical rigor with moral responsibility. Future legal professionals must be equipped not only with traditional doctrinal knowledge but also with a philosophical understanding of the limitations and possibilities of formal reasoning in normative contexts. This suggests a need for interdisciplinary approaches that integrate legal theory with moral philosophy, logic, and emerging technologies. The apparent conflict between formal logic and justice principles is not a flaw to be corrected but a fundamental characteristic of law that enables its adaptability and humanity. Recognition of these paradoxes, rather than their elimination, may be the key to developing more sophisticated and just legal systems.

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